Atomic clocks and fundamental tests in Physics

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Atomic Clocks and Fundamental Tests in Physics: Optical Frequency Standards

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OUTLINE

- Status of atomic fountains and motivations for optical clocks
- Optical clock transitions
- Optical ion clocks
- Optical lattice clocks
- Ultra stable lasers
- Measuring Optical Frequencies
- Optical Links
- Fundamental tests
Introduction
Principle of atomic clocks (1)

Goal: deliver a signal with stable and universal frequency

Bohr frequencies of unperturbed atoms are expected to be stable and universal

\[ h\omega_{ef} = h\nu_{ef} = E_e - E_f \]

Building blocks of an atomic clock

macroscopic oscillator

\[ \omega(t) = \omega_{ef} \times \left(1 + \varepsilon + y(t)\right) \]

\( \varepsilon \): fractional frequency offset

**Accuracy**: overall uncertainty on \( \varepsilon \)

\( y(t) \): fractional frequency fluctuations

**Stability**: statistical properties of \( y(t) \), characterized by the Allan variance \( \sigma_y^2(\tau) \)

Can be done with microwave or optical frequencies, with neutral atoms, ions or molecules
Principle of atomic clocks (2)

How to probe the atomic transition:

The two main parameters:

1. the atomic quality factor: \( Q_{at} = \frac{\nu_{ef}}{\Delta \nu} \propto \nu_{ef} T \)

2. fluctuations of the measured transition probability for integration time \( T_c \): \( \sigma_{\delta P} \)

Scaling of the fractional frequency instability:

\[ \sigma_y(\tau) \propto \frac{\sigma_{\delta P}}{Q_{at}} \sqrt{\frac{T_c}{\tau}} \]

Example: optimized Ramsey interrogation

\[ \sigma_y(\tau) = \frac{2 \sigma_{\delta P}}{\pi Q_{at}} \times \sqrt{\frac{T_c}{\tau}} \]
Atomic fountain clocks

\[ 133\text{Cs levels (}^{87}\text{Rb similar)} \]

- Ramsey fringes

Atomic quality factor:

\[ Q_{\text{at}} = \frac{\nu_{\text{ef}}}{\Delta \nu} \approx 9.8 \times 10^{15} \]

Best frequency stability (~ Quantum Projection Noise limited): \(1.6\times10^{-14} @1s\)

Best accuracy: \(4\times10^{-16}\)

~ 10 fountains in operation (LNE-SYRTE, PTB, NIST, USNO, JPL, METAS, INRIM, NPL, USP, NICT, ...)

with an accuracy a few \(10^{-15}\) and \(<10^{-15}\) for a few of them.
LNE-SYRTE ATOMIC CLOCK ENSEMBLE

- **H-maser**
- **Cs, µW**
- **Rb, Cs, µW**
- **Sr, opt**
- **Hg, opt**
- **FO1 fountain**
- **FO2 fountain**
- **Cryogenic sapphire Osc.**
- **Macroscopic oscillator**
- **Phaselock loop** \( \tau \sim 1000 \text{ s} \)
- **FOM transportable fountain**
- **Optical lattice clock**

Systèmes de Référence Temps-Espace
Cesium fountain comparisons

Routine comparison down to the $10^{-16}$ level.

Cumulate hundreds of days of comparison supporting this uncertainty budget.

Not perfect though:

- 1 to 1.5 $\sigma$ frequency difference
- Some flickering at the $\sim 4 \times 10^{-16}$ level
Rb/Cs frequency comparison since 1998

Weighted least square fit by a constant

Chi2-goodness of fit
Q = 0.75

\[ \nu_{\text{Rb}} \]

- data and error bars are perfectly consistent
- new value (dual FO2 + FO1)
- previous points: FO2-Rb comparison with FOM or FO1

Note: Errors bars are type B dominated

BIPM CCTF recommended value (based on LNE-SYRTE 2002 data):
\[ \nu_{\text{Rb}}(\text{CCTF}) = 6\,834\,682\,610.904\,324\,(21)\ \text{Hz} \]

- These data and other fountain measurements have been and are still used for fundamental physics tests:
  - Stability of fundamental constants, Local Position Invariance, Local Lorentz Invariance, Isotropy of space
Other contributions of atomic fountains

- Absolute frequency measurements of optical frequencies, including in remote locations with the transportable fountain.
  - Sr, Hg lattice clocks at SYRTE.
  - $^{40}$Ca$^+$ in Innsbruck with transportable fountain FOM.
  - H(1S-2S) at MPQ-Garching FOM.
- Support to the development of the cold atom space clock PHARAO/ACES.
- Ground segment of PHARAO/ACES during the mission.
- Remote comparisons. Contribution to testing the performance of satellite T&F transfer systems.
Contributions of atomic fountains to timekeeping

Calibration of TAI by atomic fountains

Over the last 3 years, LNE-SYRTE fountains have made ~ 50% of all formal calibration reports to BIPM

- 14/28 formal reports in 2007
- 23/45 in 2008

One report corresponds typical to a quasi continuous measurement of a H-maser frequency for 15 to 30 days

\[ u_B \sim 4.5 \times 10^{-16} \]
\[ u_A \sim 1 \times 10^{-16} \]
\[ u_{\text{link/maser}} \sim 1.5 \times 10^{-16} \]

USNO is developing several $^{87}\text{Rb}$ fountains for the GPS master clock ensemble

\[ \Rightarrow \text{One can safely predict a long lifetime to atomic fountains} \]

\[ \text{However, reaching accuracy} < 10^{-16} \text{ with atomic fountain seems unlikely} \]
Motivations for developing optical clocks: Stability

Stability of an atomic clock:

$$\sigma_y(\tau) \propto \frac{\sigma_\delta P}{Q_{at}} \sqrt{\frac{T_c}{\tau}}$$

Quantum limited stability:

$$\sigma_y(\tau) = \frac{1}{\pi Q_{at}} \times \frac{1}{\sqrt{N_{det}}} \times \sqrt{\frac{T_c}{\tau}}$$

Microwave transition: $\nu_{ef} \sim 10$ GHz, Optical transition: $\nu_{ef} = c/\lambda \sim 10^{14}$ Hz

**Example #1:** cycle $T_c \sim 1$s, linewidth $\sim 1$ Hz et $\sigma_\delta P \sim 10^{-4}$:

Microwave: $\sigma_y = 10^{-14} @1s$

Optical: $\sigma_y = 10^{-18} @1s$

In practice, best reported stabilities

$\sigma_y = \sim 10^{-15} @1s$

**Example #2:** cycle $T_c \sim 1$s, linewidth $\sim 1$ Hz et $N_{det} \sim 1$ (1 atom!):

Optical: $\sigma_y = 3 \times 10^{-15} @1s$
Motivations for developing optical clocks: Accuracy

- Better stability implies better resolution to evaluate systematic shifts
- Some systematic shifts are smaller as compared to microwave clocks

<table>
<thead>
<tr>
<th>Effect</th>
<th>Uncertainty FO2 (x 10^{-16})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectral purity of interrogation oscillator, Microwave leaks</td>
<td>+/- 0.1</td>
</tr>
<tr>
<td>Ramsey and Rabi pulling</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Second order Zeeman</td>
<td>+/- 0.3</td>
</tr>
<tr>
<td>Cold collisions</td>
<td>+/- 2.0</td>
</tr>
<tr>
<td>Blackbody radiation</td>
<td>+/- 0.6</td>
</tr>
<tr>
<td>Residual 1st order Doppler</td>
<td>&lt; 3</td>
</tr>
<tr>
<td>Quantum motion (&quot;microwave recoil&quot;)</td>
<td>&lt; 1.4</td>
</tr>
<tr>
<td>Quadratic sum</td>
<td>&lt; 4</td>
</tr>
</tbody>
</table>

Go down when $Q_{at}$ increases

Do not go down when $Q_{at}$ increases but better atom/transition can be chosen

Effects of external variables independent of the frequency:

$$\delta \omega_{Doppler} = \frac{k \cdot \vec{v} \sim \omega \cdot \frac{v}{c}}$$

Accuracy with free falling Ca atoms (NIST, PTB): $\sim 10^{-14}$


$\Rightarrow 10^{-17}$ or better is within reach provided a drastic solution is found for the effects of external motion
Motional effects on free atoms

Two level atom absorbing a photon

- Energy of the atom in $g$ and $e$ as a function of its momentum $\hbar k_{at}$:

$$E_g = \frac{\hbar^2 k_{at}^2}{2m} \quad E_e = \hbar \omega_{at} + \frac{\hbar^2 k_{at}^2}{2m}$$

- Atom absorbs a photon with energy $\hbar \omega$ and momentum $\hbar k$

- Total momentum conservation implies that:

$$k_{at} \rightarrow k_{at} + k$$

- Total energy conservation implies that:

$$\hbar \omega + \frac{\hbar^2 k_{at}^2}{2m} = \hbar \omega_{at} + \frac{\hbar^2 (k_{at} + k)^2}{2m}$$

The resonant frequency for absorption is:

$$\hbar \omega = \hbar \omega_{at} + k \cdot \frac{\hbar k_{at}}{m} + \frac{\hbar^2 k^2}{2m}$$

1st order Doppler recoil
Motional effects on confined atoms

Consider an atom spatially confined to much less than the wavelength of the incoming photon

\[ \Delta x_{at} \ll \frac{\lambda}{4\pi} \]

In momentum space, the atomic wave-function is such that:

\[ \Delta p_{at} \cdot \Delta x_{at} > \hbar/2 \quad \text{or} \quad \Delta k_{at} \cdot \Delta x_{at} > 1/2 \]

This implies that:

- the size of the wave-function in momentum space is large compared to the photon momentum
- The shift in momentum space implied by momentum conservation induces a minor modification of the wave-packet
- Small shift of the resonant frequency when the energy conservation is applied
Optical Clock Transitions
Requirements for an optical clock transition

- Electric dipole allowed transitions typically have natural linewidth ~ or > 1 MHz.
  - $Q_\text{at} \sim 10^8$ or less $\Rightarrow$ not interesting ($Q_\text{at} \sim 10^{10}$ in atomic fountains)

- For clocks, long lived states and “forbidden” transitions are necessary.

- Several mechanisms are possible. For instance:
  - Electric quadrupole, electric octupole transitions
  - Intercombination transitions
  - 2 or multi-photon transitions
  - ...

- Blackbody radiation shift must be considered since the large $\sim 10^4$-$10^5$ reduction factor between optical and microwave transitions does not apply to this shift.
Hamiltonian for an atom in the presence of an external electromagnetic field

\[ H = H_{\text{at}} - \sum_i \frac{q_i}{2m_i} \left[ \vec{P}_i \cdot \vec{A}(\vec{R}_i) + \vec{A}(\vec{R}_i) \cdot \vec{P}_i \right] + \sum_i \frac{q_i^2}{2m_i} \left[ \vec{A}(\vec{R}_i) \cdot \vec{A}(\vec{R}_i) \right] \]

**Momentum and position operator for particle i in atom**

**Vector potential of the external field**

External field not to strong

Traveling wave

\[ \vec{A}(\vec{r}) = \vec{A}_0 e^{i(k \vec{r} - \omega t)} \]

Wavelength long compared to size of atom

\[ k \cdot R_i \ll 1 \quad \lambda \gg R_i \]

**Dipolar approximation**

\[ e^{i \vec{k} \cdot \vec{R}_i} \approx 1 \]

**Electric dipole transitions E1**

\[ \Delta J = 0, \pm 1 \quad \Delta \gamma \neq 0 \quad (J = 0 \neq 0) \]

**Electric quadrupole E2**

\[ \Delta J = 0, \pm 1, \pm 2 \quad \Delta \gamma = 0, 0, \pm 1 \quad (J = 0 \neq 0, 1; \quad \frac{1}{2} \neq \pm \frac{1}{2}) \]

**Electric octupole E3**

\[ \Delta J = 0, \pm 1, \pm 2, \pm 3 \quad \Delta \gamma = 0, 0, \pm 1, \pm 2 \quad (0 \neq 0, 1, 2; \quad \frac{1}{2} \neq \pm \frac{1}{2}, \frac{3}{2}; \quad 1 \neq \pm 1) \]
E2 and E3 transitions used in optical clocks

**199Hg**

- **E2** transition: $\tau \sim 90 \text{ ms}$, $\Delta \nu \sim 1.8 \text{ Hz}$
- Frequency: $\nu = 1.06 \times 10^{15} \text{ Hz}$
- Wavelength: $\lambda = 282 \text{ nm}$

**88Sr**

- **E2** transition: $\tau \sim 400 \text{ ms}$, $\Delta \nu \sim 0.4 \text{ Hz}$
- Frequency: $\nu = 4.45 \times 10^{14} \text{ Hz}$
- Wavelength: $\lambda = 674 \text{ nm}$

**171Yb**

- **E2** transition: $\tau \sim 51 \text{ ms}$, $\Delta \nu \sim 3.1 \text{ Hz}$
- Frequency: $\nu = 6.88 \times 10^{14} \text{ Hz}$
- Wavelength: $\lambda = 435.5 \text{ nm}$

**171Yb**

- **E3** transition: $\tau \sim 10 \text{ years}$, $\Delta \nu \sim 0.5 \text{ nHz}$
- Frequency: $\nu = 6.42 \times 10^{14} \text{ Hz}$
- Wavelength: $\lambda = 467 \text{ nm}$
**Intercombination transitions**

- Atoms with 2 electrons on the outermost shell: Alkaline-earth and alkaline-earth-like atoms
- Electronic structure splits in singlet states with $S=0$ and triplet states with $S=1$
- Selection rule for E1 transition: $\Delta S=0$

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Mass</th>
<th>Abundance</th>
<th>Spin</th>
<th>Mag Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{87}\text{Sr}$</td>
<td>86.908884</td>
<td>7.00%</td>
<td>9/2</td>
<td>-1.093</td>
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<td>$^{85}\text{Sr}$</td>
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<tr>
<td>$^{86}\text{Sr}$</td>
<td>85.913430</td>
<td>0.56%</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**Gross structure**

<table>
<thead>
<tr>
<th>Singlet</th>
<th>Triplet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1\text{S}_0$</td>
<td>$^3\text{P}_1$</td>
</tr>
<tr>
<td>$^3\text{P}_0$</td>
<td>$^3\text{P}_2$</td>
</tr>
<tr>
<td>$^3\text{P}_1$</td>
<td>$^1\text{P}_1$</td>
</tr>
</tbody>
</table>

**Fine structure interaction**

(L.S, S.S couplings +QED)

- Hyperfine interaction
- Nuclear spin $I=9/2$ (fermion only)

Alkaline-earth and alkaline-earth like atoms/ions considered for optical clocks

Periodic table from NIST website
Intercombination transitions used in optical clocks

\[ ^{87}\text{Sr} \]
- \( \tau \approx 167\,\text{s} \)
- \( \Delta \nu \approx 1\,\text{mHz} \)
- \( \nu = 4.29 \times 10^{14}\,\text{Hz} \)
- \( \lambda = 698\,\text{nm} \)

\[ ^{171}\text{Yb} \]
- \( \tau \approx 15\,\text{s} \)
- \( \Delta \nu \approx 10\,\text{mHz} \)
- \( \nu = 5.18 \times 10^{14}\,\text{Hz} \)
- \( \lambda = 578\,\text{nm} \)

\[ ^{199}\text{Hg} \]
- \( \tau \approx 1.5\,\text{s} \)
- \( \Delta \nu \approx 100\,\text{mHz} \)
- \( \nu = 1.13 \times 10^{15}\,\text{Hz} \)
- \( \lambda = 265.6\,\text{nm} \)

\[ ^{27}\text{Al}^+ \]
- \( \tau \approx 377\,\text{s} \)
- \( \Delta \nu \approx 0.4\,\text{mHz} \)
- \( \nu = 1.12 \times 10^{15}\,\text{Hz} \)
- \( \lambda = 267\,\text{nm} \)
Other schemes with alkaline-earth-like atoms

Sr, Yb, Hg bosonic isotopes have no nuclear spin → No hyperfine mixing

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<tr>
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<td>0</td>
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<td>-1.093</td>
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<td>$^{88}\text{Sr}$</td>
<td>87.905619</td>
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</tbody>
</table>

Gross structure

Fine structure interaction (L.S, S.S couplings +QED)

Quenching with a static magnetic field


Use a 2 photon transition

Other interesting transitions

Nuclear transition in Thorium $^{229}\text{Th}$

- Generally, nuclear excited states are very far apart at the scale of the optical spectrum
- In $^{229}\text{Th}$ nucleus, an excited state lies very close to the ground state
- The two isomers have different nuclear magnetic moment
- The transition between the two states is $M1$ (magnetic dipole transition)
- Transition frequency is known from high-precision $\Gamma$-ray spectroscopy
  \[ (7.6 \pm 0.1) \text{ eV} \Leftrightarrow (163 \pm 11) \text{ nm} \]
- The state of ionization of $^{229}\text{Th}$ atom can be chosen to ease laser cooling and detection. Triply ionized Th is foreseen as a good choice
- Very interesting for fundamental physics given the different nature of this transition
- Transition not observed directly so far

Prospects for a Nuclear Optical Frequency Standard based on Thorium-229
E. Peik et al., arXiv:0812.3548v2

Nuclear laser spectroscopy of the 3.5 eV transition in Th-229

Two photon transition in Ag

Blackbody radiation shift

Spectral density of blackbody radiation at 300 K

Peak: 1.7×10^{13} Hz (λ~17µm)

\[
\rho(\omega, T) = \frac{\hbar \omega}{\exp\left(\frac{\hbar \omega}{k_B T}\right) - 1} \times \frac{\omega^2}{\pi^2 c^3}
\]

\[
\langle E^2 \rangle = K_E \left(\frac{T}{T_0}\right)^4, \quad \langle B^2 \rangle = \frac{\langle E^2 \rangle}{c^2}
\]

\[T_0 = 300 \text{ K et } K_E \simeq (831.9 \text{ V.m}^{-1})^2\]

Most of the effect is given by the RMS value of the Stark shift (\(\propto E^2\)) \(\Rightarrow\) The shift varies as \(T^4\)

- It is mostly preferred to avoid cryogenics
  - Already very complex experiments
  - Long term operation required for applications

- A priori, no large gain compared to fountains
  - The shift is differential in microwave clocks

- The shift is hardly controlled to \(<10^{-2}\)

### Table: Fractional shift due to blackbody radiation @ 300 K

<table>
<thead>
<tr>
<th>Atom</th>
<th>Fractional shift due to blackbody radiation @ 300 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cs (µW)</td>
<td>-1.7×10^{-14}</td>
</tr>
<tr>
<td>Rb (µW)</td>
<td>-1.3×10^{-14}</td>
</tr>
<tr>
<td>Sr</td>
<td>-5.5×10^{-15}</td>
</tr>
<tr>
<td>Ca, Yb</td>
<td>-2.6×10^{-15}</td>
</tr>
<tr>
<td>H(1S-2S)</td>
<td>-4.2×10^{-16}</td>
</tr>
<tr>
<td>Mg</td>
<td>-3.9×10^{-16}</td>
</tr>
<tr>
<td>Hg</td>
<td>-1.6×10^{-16}</td>
</tr>
<tr>
<td>Hg^+</td>
<td>-7.5×10^{-17}</td>
</tr>
<tr>
<td>Al^+</td>
<td>-8×10^{-18}</td>
</tr>
</tbody>
</table>
Optical Ion Clocks
Ion trapping: Field Configuration in a RF trap

- Use the force exerted on a charged particle by an electric field

- Particle at rest at the trap center: $E=0 \iff 1^{st}$ derivative of potential $= 0$
- Simplest potential:
  \[
  \Phi(\vec{r}) = \frac{U}{2} (\alpha x^2 + \beta y^2 + \gamma z^2)
  \]

- The potential must satisfy Laplace equation: $\alpha + \beta + \gamma = 0$.
  - Implies that confinement is not possible in all dimensions with a static potential
- What if the trapping potential is sinusoidally varying in time?
  \[
  \Phi(\vec{r}, t) = U \cos(\omega_R t) \frac{1}{2} (\alpha x^2 + \beta y^2 + \gamma z^2)
  \]

- Note: An other possibility for stable trapping is to use a large static magnetic field $\rightarrow$ Penning trap

H. Dehmelt, Rev. Mod. Phys. 62, 525 (1990)
W. Paul, Rev. Mod. Phys. 62, 531 (1990)
Ion trapping: Practical realization of the field

- Equipotential surfaces are hyperboloids
- Ideally, one should use electrodes with hyperboloid shapes

Practically, simpler geometries are often used for optical clocks and Quantum Information Processing experiments, without much drawbacks

- Ions are 1 or few. They are laser cooled. They only see a small region near the trap center. Not very sensitive to higher order terms in the field

Generally, a superposition of static and varying potential can be applied

\[ \Phi(\vec{r}, t) = U \frac{1}{2}(\alpha x^2 + \beta y^2 + \gamma z^2) + \tilde{U} \cos(\omega_{rf}t) \frac{1}{2}(\alpha' x'^2 + \beta' y'^2 + \gamma' z'^2) \]
Ion trapping: Pictures of some traps

- Many other groups: NPL, NRC,... + QIP research groups
**Ion trapping: classical motion**

**Classical equation of motion: Mathieu equations**

\[
\ddot{u}_i(t) + \left( a_i + 2q_i \cos(\omega_{rf} t) \right) \frac{\omega_{rf}^2}{4} u_i = 0, \quad i = x, y, z
\]

with \( a_x = \frac{4ZeU\alpha}{m\omega_{rf}^2}, \quad q_x = \frac{2Ze\tilde{U}\alpha'}{m\omega_{rf}^2} \)

(and similarly: \( a_y, a_z, q_y, q_z \))

**Stable solutions exist where the trajectory is bounded:**

- **secular motion**
- **micro motion**

\[
u_i(t) \simeq u_{i0} \times \cos(\omega_i t + \varphi_{i0}) \left[ 1 + \frac{q_i}{2} \cos(\omega_{rf} t) \right]
\]

\[
\omega_i^2 = \frac{\omega_{rf}^2}{4} \left( a_i + \frac{q_i^2}{2} \right)
\]

for \( |a_i|, q_i^2 \ll 1 \)

**Stability diagram**

- **Important application:** Mass spectrometer

![Stability diagram image](image-url)
Ion trapping: quantum motion

The Hamiltonian looks like:

$$H^{(m)}(t) = \frac{P^2}{2m} + \frac{m}{2} W(t) X^2$$

$$W(t) = \frac{\omega_{rf}^2}{4} [a_x + 2 q_x \cos(\omega_{rf} t)]$$

For $|a_x|, q_x^2 \ll 1 \Rightarrow$ Effective potential approximation $\Rightarrow$ Approximated solutions can be written as:

$$\psi(x,t) \simeq \phi(x,t) \times \exp\left\{... \cos(\omega_{rf} t) ...\right\}$$

Breathing of the wavefunction at $\omega_{rf} \Leftrightarrow$ micromotion

- where an effective time-independent harmonic oscillator Hamiltonian applies to

$$\begin{align*}
H^{(m)}(t) &\quad (|a_x|, q_x^2) \ll 1 \\
\psi(x,t) &\quad H_{eff} = \frac{P^2}{2m} + \frac{1}{2} m \omega_x^2 X^2
\end{align*}$$

- where $\omega_x$ is the classical secular frequencies

Complete formal analogy to a static harmonic potential. The effective Hamiltonian can be written as:

$$H_{eff} = \hbar \omega_x \left( a^\dagger a + \frac{1}{2} \right)$$

- However, the complete states are not energy eigenstates. The micromotion periodically change the total kinetical energy of the state.
Consider an ion with two internal energy levels $e$ and $g$

$$|e\rangle \quad H^{(e)} = \hbar \omega_{eg} \left( |e\rangle \langle e| - |g\rangle \langle g| \right)$$

$\hbar \omega_{eg} = E_e - E_g$

Energy levels of the trapped ion

$$H^{(e)} + H^{(m)}(t) \quad H^{(e)} + H^{(m)^*}$$

Trapped ion interacting with light propagating along $x$

$$H^{(i)}(t) = (\hbar/2) \Omega (|g\rangle \langle e| + |e\rangle \langle g|)$$

$$\times [e^{i(k\hat{x} - \omega t + \phi)} + e^{-i(k\hat{x} - \omega t + \phi)}].$$

$$H^{(e)} + H^{(m)}(t) + H^{(i)}(t) \quad (|a_x|, q_x^2) \ll 1 \quad \eta \ll 1$$

$$H^{(e)} + H^{(m)} + H^{(i)}(t)$$

where we have introduced the Lamb-Dicke parameter:

$$H^{(i)}(t) = \frac{\hbar}{2} \Omega_0 \langle e\rangle \langle g| \exp \left\{ i\eta (ae^{-i\omega_xt} + a^\dagger e^{i\omega_xt}) \right\} e^{i(\phi - \delta t)} + h.c.$$  

Lamb-Dicke regime

$$\eta \sqrt{\langle (a + a^\dagger)^2 \rangle} \ll 1$$
Parameter of a $^{199}\text{Hg}^+$ ion trap

Nominally, no static voltage: $a_x = a_y = a_z = 0$

Ring inner diameter: 0.8 mm

Drive frequency: 10 MHz

Drive amplitude: 1 kV

Radial secular frequencies: $\nu_x, \nu_y \sim 1$ MHz

Axial secular frequency: $\nu_z \sim 2$ MHz

$q_x^2, q_y^2 \sim 0.1$

$q_z^2 \sim 0.5$

Wavelength of the clock transition: $\lambda = 282$ nm

Recoil frequency of the clock transition: $\nu_R = 13$ kHz

Lamb-Dicke parameter: $\eta^2 \sim 0.01, \eta \sim 0.1$

Extension of the vibrational ground state wavefunction: $z_0 \sim 5$ nm

The trap is loaded by e$^-$ impact ionization or by photoionization of a vapor

The same ion can be kept for months at 4 K, for Hg$^+$
at room temperature, for Yb$^+$
Doppler cooling and fluorescence detection

Doppler cooling

- Transition with decay rate faster than trap frequencies
- Similar to free space, similar cooling limit
- Laser tuned \( \sim \Gamma/2 \) to the red of the cooling resonance

\[
T_{\text{min}} = \frac{\hbar \sqrt{1 + s}}{4 k_B \Gamma} (1 + \xi)
\]

Hg+: cooling wavelength: 194 nm
\( \Gamma = 1/\tau; \tau \sim 2 \text{ ns} \implies T_{\text{min}} \sim 1.7 \text{ mK} \)

- Average vibrational quantum number: \( \langle n_i \rangle \sim 35 \)
- RMS size of motion:
  \( \Delta z^2 = z_0^2 \left(2\langle n_i \rangle + 1\right) \implies \Delta z \sim 355 \text{ nm} \)

Fluorescence detection

- When cooling light is on, the ion is scattering photons
- This is providing a way to detect the ion
Temporal sequence for probing the clock transition

Cooling

\[ \begin{align*}
&\text{\(^{2}P_{1/2}\)} \\
\text{\(F=1\)} & \quad \tau(\text{\(^{2}P_{1/2}\)}) \sim 2\,\text{ns} \\
\text{\(F=0\)} & \quad 6.9\,\text{GHz} \\
&\text{\(^{2}D_{5/2}\)} \\
\text{\(F=3\)} & \quad \tau(\text{\(^{2}D_{5/2}\)}) \sim 90\,\text{ms} \\
\text{\(F=2\)} & \quad \text{F=1} \\
\text{\(^{2}S_{1/2}\)} & \quad \text{F=0} \\
& \quad 40.5\,\text{GHz}
\end{align*} \]
Temporal sequence for probing the clock transition

Pumping

$^{2}P_{1/2}$ $^P_{1/2} \sim 2 \text{ ns}$

$^{2}D_{5/2}$ $^{D}_{5/2} \sim 90 \text{ ms}$

$^{2}S_{1/2}$ $^{S}_{1/2}$

6.9 GHz

40.5 GHz
Temporal sequence for probing the clock transition

Clock transition excitation

\[ ^2P_{1/2} \]  
\( F=1 \quad \tau(^2P_{1/2}) \approx 2 \text{ ns} \)  
\( F=0 \)  
6.9 GHz

\[ ^2D_{5/2} \]  
\( F=3 \quad \tau(^2D_{5/2}) \approx 90 \text{ ms} \)  
\( F=2 \)  
40.5 GHz

\[ ^2S_{1/2} \]  
\( F=1 \)  
\( F=0 \)
Temporal sequence for probing the clock transition

Detection

Case #1:
Fluorescence is detected $\Rightarrow$ Ion is in the ground state.

Case #2:
No fluorescence is detected $\Rightarrow$ Ion is in the excited state.
Temporal sequence for probing the clock transition

If case #2: Depumping

Case #2: Wait for fluorescence to reappear due to spontaneous decay or depumping of the excited state.
Spectroscopy of the optical clock transition in $^{199}$Hg$^+$

**Carrier and secular sidebands**

- Not strongly in the Lamb-Dicke regime
- Resolved sideband regime

Recoilless optical absorption and Doppler sidebands of a single trapped ion

Narrow line spectroscopy of the carrier

- Line width: 6.5 Hz at $1.06 \times 10^{15}$ Hz
- $Q_{at} \approx 1.6 \times 10^{14}$
- Quantum limited detection
- Clock upper state lifetime: 90 ms $\Leftrightarrow$ 2 Hz
- Quantum limited stability $\sim 10^{-15} \@ 1$s

Recoilless optical absorption and Doppler sidebands of a single trapped ion
Cooling to the ground state of (secular) motion

Sideband cooling
- Narrow enough transition + narrow enough excitation laser
- \( \rightarrow \) motional sidebands are resolved
- Laser excitation of the red sideband
- Spontaneous decay
- \( \rightarrow \) accumulation in the vibrational ground state
- Cooling limit: off-resonant excitation of the carrier

\[
k_B T_{\text{min}} = \hbar \omega_i \langle n_i \rangle \sim \hbar \omega_i \times \frac{\Gamma^2}{4 \omega_i^2}
\]

- Raman sideband cooling is similar with
  - Raman transition
  - Repumper

- 2 (or more) ions in a linear trap
  - Sideband spectrum shows eigenmodes of motion
  - Center of mass motion, breathing mode,…
  - Sideband cooling can be applied
  - Sympathetic cooling of 1 ion by the other through the strong coulomb interaction
Detection and spectroscopy using quantum logic

Assume the following situation:
- 2 ions in a linear trap: Spectroscopy (clock) ion (S: Al\(^{+}\)) + Logic ion (L: Be\(^{+}\))
- System cooled to the vibration ground state (sympathetic Raman sideband cooling with the logic ion)
- Usual probe pulse on the clock transition of the S ion

The scheme is a way to detect the out-coming state of the S ion using a quantum logic gate to transfer the information to the L ion:
- Vibrational spectrum is present on both L and S transitions and can be excited either by L or S excitation

Fluorescence means "detection in the ground state of the S ion"
No Fluorescence means "detection in the excited state of the S ion"
Frequency comparisons between ion clocks

Comparison between two $^{171}$Yb$^+$ clocks on the quadrupole transition

Stability: $\sim 1.6 \times 10^{-14}$ @1s

Chou et al., PRL 104, 070802 (2010)

Comparison between two Al$^+$ quantum logic clocks

Stability: $\sim 2.8 \times 10^{-15}$ @1s

Chou et al., PRL 94, 230801 (2005)
Accuracy budget in Yb+ quadrupole and Al+ clocks

### Yb+ quadrupole

<table>
<thead>
<tr>
<th>Physical Effect</th>
<th>Correction (Hz)</th>
<th>Uncertainty (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second-order Zeeman shift</td>
<td>-1.13</td>
<td>0.05</td>
</tr>
<tr>
<td>Quadrupole shift and tensorial quadratic Stark shift</td>
<td>-0.36 to -0.19</td>
<td>0.2</td>
</tr>
<tr>
<td>Scalar ac Stark shift</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>Servo error</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>Total</td>
<td>-1.43 to -1.26</td>
<td>0.31</td>
</tr>
</tbody>
</table>

+ Black body radiation shift correction (at 300 K): + 5.09(10)x10^{-16} (from theory)

Tamm et al., PRA 80, 043403 (2009)

### Al+

- Some shifts are specific to ion clock
  - Micromotion
  - Electric quadrupole shift
  - BBR and electrode heating
  - Synchronous perturbation of E field by the probe light

Chou et al., PRL 104, 070802 (2010)
Optical Lattice Clocks
**Polarizability, dipole force and dipole trapping**

**Atom in a laser field: induced dipole**

\[ E(z,t) = E \times e^{i(kz-\omega t)} \]

\[ D = \alpha(\omega) \times E \]

Laser intensity:

\[ \Pi = \frac{1}{2} \varepsilon_0 \times E^2 \]

Strontium ground state static polarizability:

\[ \frac{\alpha(\omega = 0)}{(4\pi \varepsilon_0)} = 27.6 \times 10^{-30} \ m^3 \sim 186 \ (a_0)^3 \]

**Interaction energy between induced dipole and laser field**

\[ U = -\frac{1}{4} \alpha(\omega) \times E^2 \]

Laser intensity: 10 kW.cm\(^{-2}\) \(\Rightarrow U/2\pi\hbar \sim 150 \text{ kHz, } U/k_b \sim 7.2 \mu\text{K}\)

Interaction can be used to trap atoms

- Spatial variation of laser field intensity
- \(\Rightarrow\) spatial variation of U
- \(\Rightarrow\) force on atom
- \(\Rightarrow\) widely used in quantum gases exp.,...


**This interaction is producing a light of energy levels**

- 150 kHz \(\Leftrightarrow~3\times10^{-10}\) fractional shift of the clock transition !!!
- Bad starting point for a clock?
Polarizability as a function of frequency

\[ \alpha(3P_0) \quad \alpha(1S_0) \]

Fig. 7.42. Combined partial energy-level – Grotrian diagram with absorption oscillator strengths for mercury
« Non-perturbing » dipole trap


Light shift as a function of trap wavelength

Magic wavelength $\lambda_{\text{magic}}$: polarizabilities are equal for both clock states

Light shift of the clock frequency:

Sensitivity: fractional shift vs detuning from $\lambda_{\text{magic}}$:

$\Rightarrow$ under typical conditions: $\sim 10^{-15}$ /GHz
« Non-perturbing » lattice trap

- Lamb-Dicke regime requires strong confinement
- \(\Rightarrow\) strong variation of the trapping potential over short distance
- \(\Rightarrow\) use a lattice trap formed by a standing wave

![Diagram of a lattice trap](image)

Natural unit of energy to characterize the trap depth \(U_0\): recoil energy at the trap wavelength:

\[
E_R = \frac{\hbar^2 k_l^2}{2m a_t} \quad k_l = \frac{2\pi}{\lambda_l}
\]

![Mathematical expression for trap potential](image)

\[
U(r) = \frac{U_0}{2} e^{-2r^2/\omega_0^2} (1 - \cos(2k_l z))
\]

\[
U_0 = -\frac{1}{4} |\alpha| \omega_0^2
\]

- Combines advantage of trapped ion and neutral atoms
- \(\Rightarrow\) large atom number in the Lamb-Dicke regime
Parameters of a strontium lattice trap

Magic wavelength: \( \lambda_l = 813.428(1) \text{ nm} \)

Recoil energy: 
\[
\frac{E_R}{(2\pi\hbar)} = 3.5 \text{ kHz} \\
\frac{E_R}{k_B} = 0.16 \text{ } \mu\text{K}
\]

Trap depth: \( U_0 \sim 1400 \times E_R \Leftrightarrow \sim 4 \text{ MHz } \Leftrightarrow \sim 200 \text{ } \mu\text{K} \)

- Dicke parameter

Longitudinal trap frequency: 
\[
\omega_z/2\pi = 255 \text{ kHz}
\]

Longitudinal size of vibrational ground state wavefunction:
\[
\Delta_z \sim \sqrt{\frac{\hbar}{2m\omega_z}} \approx 15 \text{ nm}
\]

Transverse trap frequency: 
\[
\omega_r/2\pi = 540 \text{ Hz}
\]

Transverse size of vibrational ground state wavefunction:
\[
\Delta_r \sim \sqrt{\frac{\hbar}{2m\omega_r}} \approx 327 \text{ nm}
\]
Temporal sequence for probing the clock transition

Loading into the dipole trap

\[ \text{laser cooling} \]

\[ \text{trap} \]

\[ \text{Blue MOT} \]

\[ (2 \text{ mK}) \]

\[ \text{Dipole trap beam} \]

\[ (w_0 = 90 \mu \text{m}, U_0 = 200 \mu \text{K}) \]

\[ \text{at} \]

\[ \lambda_{\text{trap}} \]

\[ \text{atomic drain} \]

\[ (w_0 = 50 \mu \text{m}) \]

\[ \rightarrow 10^4 \text{ atoms loaded} \]
Loading into the dipole trap

Temporal sequence for probing the clock transition

1S0 → 3P0 → 3S1 → 1P1

707 nm (7 MHz)
679 nm (1.75 MHz)

707 nm + 679 nm: repumpers ($w_0 = 200 \mu m$)

Dipole trap beam
Cooling in the dipole trap

Temporal sequence for probing the clock transition

$^1P_1 \rightarrow ^3S_1$

$^1S_0 \rightarrow ^3P$

689 nm $\Gamma = 7.6$ kHz

Dipole trap beam

689 nm: narrow line cooling
Temporal sequence for probing the clock transition

Optical pumping

$^{3}S_{1}$

$^{1}P_{1}$

$^{3}P_{1}$

$^{1}S_{0}$

$F=9/2$

Optical pumping at 689 nm
Temporal sequence for probing the clock transition

Clock transition excitation

$^1P_1 \rightarrow ^3S_1$

$^3P_0 \rightarrow ^1S_0$

$B=1\text{G}, \Delta \zeta(-9/2 \rightarrow 9/2) \approx 1 \text{kHz}$
Temporal sequence for probing the clock transition

Measure ground state

\[ ^1P_1 \rightarrow ^3S_1 \]

Dipole trap beam

Blue probe

CCD camera
Temporal sequence for probing the clock transition

$^1P_1 \rightarrow ^3S_1 \rightarrow ^3P_0 \rightarrow ^1P_1$

707 nm (7 MHz)

679 nm (1.75 MHz)

$707 \text{ nm } + 679 \text{ nm} : \text{ repumpers (}\omega_0 = 200 \mu\text{m})$
Temporal sequence for probing the clock transition

Measure excited state

$^1P_1 \rightarrow ^3S_1$

$^1S_0 \rightarrow ^3P$

compute transition probability

Dipole trap beam

Blue probe

CCD camera
Spectroscopy in the Lamb-Dicke regime

Spectrum at “high” power
- Strongly suppressed red sideband
- 95% of atoms in the longitudinal ground state
- Sideband ratio give $T_z \approx 3 \, \mu K$
- $\Delta_z = 5 \, \text{nm}$
- Strongly in Lamb-Dicke regime

Shape of sidebands
- Can be modeled with nonharmonic term + distribution of $n_x, n_y$
- And used to determine $T_r$

$$U(r) \simeq \frac{1}{2} m \omega_z^2 z^2 + \frac{1}{2} m \omega_r^2 r^2 - \frac{1}{4U_0} m^2 \omega_z^2 \omega_r^2 z^2 r^2$$
- $T_r \approx 15 \, \mu K$

Spectrum at optimized power
- Optimized for the carrier
- Strongly suppressed coupling to sidebands

A. Brusch et al. PRL 96, 103003 (2006)
Experimental resonance in a Sr optical lattice clock (JILA, Boulder).

Spectroscopy in the Lamb-Dicke regime: State of the art

- Narrow line spectrum of the carrier

\[ Q_{at} = 2.8 \times 10^{14} \]

- \( \sim 10^4 \) atoms probed simultaneously in \( \sim 1 \) s \( \Rightarrow \) quantum projection noise limited stability would be \( \sim 10^{-16} \) @1s
  - In practice: poor duty cycle (\( \sim 10\% \)) + laser noise limit to \( 10^{-15} \) @1s

*M. M. Boyd et al., Science 314, 1430 (2006)*
Measurement of the magic wavelength


- Several longitudinal states populated
- Note: Visibly distorted line shape when $\lambda \neq \lambda_{\text{magic}}$

Later: accurate measurement of the light shift

- Residual light shift: $0.5 \pm 0.5$ Hz at $500 \, E_r$
- $3 \times 10^{-7}$ in fractional units

- @ $10 \, E_r$: E1 term $\Delta_L^{(1)} = 3(3) \times 10^{-17}$

$\Rightarrow$ Close to the target accuracy

Accurate determinations of magic wavelength:

- $368,554.38(45)$ GHz $\leftrightarrow 813.428(1)$ nm


- $368,554.68(18)$ GHz

A. Brusch et al., PRL 96, 103003 (2006)

A. D. Ludlow et al., Science 319, 1805 (2008)
Hyperpolarisability

Shift of the clock transition as a function of trapping field:

\[ h \nu = h \nu^{(0)} = \frac{1}{4} \Delta \alpha(e, \omega) E^2 - \frac{1}{64} \Delta \gamma(e, \omega) E^4 - \ldots \]

- Hyperpolarizability effect of \(-4 \text{ (4)} \mu \text{Hz/} E_R^2\) \((-0.4(4) \text{ mHz for } 10 E_R\)), corresponding to a \(-1(1) \times 10^{-18}\) fractional frequency shift
- \(\Rightarrow\) This effect will not affect the clock accuracy down to the \(10^{-18}\) level

\[ A. \text{ Brusch et al. PRL 96, 103003 (2006)} \]
Comparisons between two Sr lattice clocks

Between $^{87}\text{Sr}$ and $^{88}\text{Sr}$: isotope shift of ~62 MHz allows direct comparison

- For $^{88}\text{Sr}$, multiply populated lattice sites are eliminated using a photoassociation laser to avoid collisions
- Measurement of the isotope shift $f_{^{88}\text{Sr}} - f_{^{87}\text{Sr}} = 62, 188, 138.5(1.3) \text{ Hz}$

Ultra stable light at 698 nm $\Rightarrow$ 429 THz

3D lattice configuration for $^{88}\text{Sr}$

<table>
<thead>
<tr>
<th>Contributor</th>
<th>Correction (Hz)</th>
<th>Uncertainty (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd-order Zeeman shift</td>
<td>128.61</td>
<td>0.31</td>
</tr>
<tr>
<td>Clock light shift</td>
<td>7.48</td>
<td>0.36</td>
</tr>
<tr>
<td>Lattice light shift</td>
<td></td>
<td></td>
</tr>
<tr>
<td>scalar</td>
<td>$-0.17$</td>
<td>1.07</td>
</tr>
<tr>
<td>polarization effects</td>
<td>0</td>
<td>0.012</td>
</tr>
<tr>
<td>fourth-order</td>
<td>$-0.07$</td>
<td>0.15</td>
</tr>
<tr>
<td>Blackbody shift</td>
<td>2.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Collisional shift</td>
<td>$-0.034$</td>
<td>0.3</td>
</tr>
<tr>
<td>Systematic total</td>
<td>138.22</td>
<td>1.23</td>
</tr>
</tbody>
</table>
Accuracy and systematic shifts

Most accurate lattice clock to date

- ~2 to 3 times better than atomic fountains
- ~10 times less accurate than the best ion clock ($^{27}$Al$^+$)
- Tremendous improvements in only a few years after the “non-perturbing” lattice proposal came out

<table>
<thead>
<tr>
<th>Contributor</th>
<th>Correction ($10^{-16}$)</th>
<th>Uncertainty ($10^{-16}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lattice Stark (scalar/tensor)</td>
<td>-6.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Hyperpolarizability (lattice)</td>
<td>-0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>BBR Stark</td>
<td>52.1</td>
<td>1.0</td>
</tr>
<tr>
<td>ac Stark (probe)</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>First-order Zeeman</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Second-order Zeeman</td>
<td>0.2</td>
<td>0.02</td>
</tr>
<tr>
<td>Density</td>
<td>8.9</td>
<td>0.8</td>
</tr>
<tr>
<td>Line pulling</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>Servo error</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Second-order Doppler</td>
<td>0</td>
<td>&lt;&lt;0.01</td>
</tr>
<tr>
<td>Systematic total</td>
<td>54.9</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Ultra Stable Lasers for Optical Clocks
Important applications of ultra stable lasers

- Optical clocks and their applications (timekeeping, General Relativity tests, stability of fundamental constants,…).

- Tests of Relativity (Local Lorentz Invariance, Michelson-Morley,…).

- Generation of ultra low phase noise microwave signals and their applications (atomic fountains, ultra high resolution VLBI,…).

- Transfer of stable frequencies by fiber networks.

- Gravitational wave detection (VIRGO, LIGO,… and LISA).

- Deep space frequency transfer.

- …
Ultra stable Fabry-Perot cavities

- Currently, the reference method for laser stabilization.

- A laser is stabilized with high bandwidth to a mode of an ultra stable Fabry-Perot cavity.
  - The physical length of the cavity is defining the laser frequency.
    - $L = 0.1 \text{ m}$ and target fractional stability of $10^{-15}$ $\Rightarrow$ dimensional stability of $0.1 \text{ fm}$ (charge radius of a proton is $\sim 0.8 \text{ fm}$).

- Designs have been continuously improved to cope with:
  - Temperature sensitivity: length changes through material expansion
  - Acceleration sensitivity: length changes under mechanical stress
  - Thermal noise: length fluctuations due to Brownian motion of parts
  - Electronic noises and drifts
  - ...

- Free spectral range: $c/2L$
- Finesse: $\text{FSR}/\text{FWHM}$
Deformation of a cylinder.
- Sensitivity: $2 \times 10^{-9}/(\text{m.s}^{-2})$

\[ \frac{\Delta L}{L} = -\rho \frac{a L}{2E} \approx 2 \times 10^{-8} \]
- $L = 0.1 \text{m}$
- $E \approx 7 \times 10^{10} \text{Pa (ULE)}$
- $\rho \approx 2 \times 10^3 \text{kg/m}^3$
- $a = g = 9.81 \text{ m.s}^{-2}$

Vibration level in a laboratory.
- Measured with a commercial seismometer

- Need to reduce sensitivity
- Need to reduce vibrations
Basic concepts for low vibration sensitivity

- What matters is the optical length $L$ between the two mirrors.
- Find geometry and support scheme which minimize this quantity.

Poisson effect
Making use of symmetries?

- Ideal case: Length of interest is insensitive to transverse acceleration to first order
- Real case: the optical axis does not coincide with the mechanical axis
  - There is first order sensitivity
  - Minimizing mirror tilt is important

\[ L' = K_1 a + K_2 a^2 \]

Due to symmetry: \( K_1 = 0 \)
Example of a vertical cavity design

- FEM simulations with two different software packages
- Analysis of the sensitivity in all directions
- Aspect ratio chosen to minimize mirror tilt
- Also taken into account
  - Symmetry breaking by the support points
  - Deformation at the support points
  - Impact of modeling of support points
  - Impact of meshing
  - ...

<table>
<thead>
<tr>
<th>Sensitivity [1/m.s^{-2}]</th>
<th>FEM</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertical</td>
<td>&lt; 5x10^{-12}</td>
<td>~3.5x10^{-12}</td>
</tr>
<tr>
<td>horizontal</td>
<td>~2x10^{-12}</td>
<td>~1.4x10^{-11}</td>
</tr>
</tbody>
</table>
Horizontal cavity designs and other designs

B. Young et al., PRL 82, 3799 (1999)


C. T. Taylor et al., RSI 66, 955 (1995)


Many other groups: NRC, MPQ, Tokyo Univ.,...
Vibration isolation

- Home made passive (springs, rubber bands,...)
- Commercial devices available
- Passive
  - Smart suspension with negative stiffness mechanism
  - Low eigenfrequencies in a reasonable volume
- Active
  - Senses vibration with piezo accelerometers
  - Feed back to actuators

- Mechanical perturbations are also transmitted through acoustic noise
  - ➔ acoustic enclosure

Thermal noise (1)

Due to Brownian motion of mechanical parts

- Can be calculated using the Fluctuation-Dissipation Theorem

\[ H = \frac{X}{F} \]

- 3 contributions: spacer, mirror substrate and coatings

\[ G_{\text{spacer}}(f) = \frac{4k_B T}{\omega} \frac{L}{3\pi R^2 E} \phi_{\text{spacer}} \]

\[ G_{\text{mirror}}(f) = \frac{4k_B T}{\omega} \frac{1 - \sigma^2}{\sqrt{\pi} E \omega_0} \phi_{\text{sub}} \left( 1 + \frac{2}{\sqrt{\pi}} \frac{1 - 2\sigma}{1 - \sigma} \frac{\phi_{\text{coat}}}{\phi_{\text{sub}}} \right) \]

- Dominant contribution at low frequency: flicker noise in position

- Q factor of some materials:

<table>
<thead>
<tr>
<th>Material</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>ULE glass</td>
<td>6x10^4</td>
</tr>
<tr>
<td>Zerodur</td>
<td>3x10^3</td>
</tr>
<tr>
<td>Fused Silica</td>
<td>1x10^6</td>
</tr>
</tbody>
</table>
Thermal noise (2)

Example: cavity with \(L=100\text{mm}, R=55\text{mm}, \text{ULE glass}, w_0 = 264 \ \mu\text{m}\)

\[
\sqrt{G_{\text{spacer}}(f)} = 1.5 \times 10^{-18} \text{ m.Hz}^{-1/2} \quad \sqrt{G_{\text{substrate}}(f)} = 3.7 \times 10^{-17} \text{ m.Hz}^{-1/2} \quad \sqrt{G_{\text{coating}}(f)} \sim 1.8 \times 10^{-17} \text{ m.Hz}^{-1/2}
\]

- Corresponding fractional frequency noise PSD and instability:

\[
S_y(f) = \frac{1}{L^2} \times [2 \times G_{\text{spacer}}^2(f) + 2 \times G_{\text{substrate}}^2(f) + 2 \times G_{\text{coating}}^2(f)] \quad \sigma_y(\tau) = \sqrt{2 \ln 2 \times S_y(f = 1 \text{ Hz})}
\]

- Dominated by substrate: \(\sigma_y(\tau) = 6.8 \times 10^{-16}\)
- Same example with Fused Silica substrate
  - Dominated by coating: \(\sigma_y(\tau) = 3.2 \times 10^{-16}\)
  - Limit from spacer + substrate is: \(\sigma_y(\tau) = 1.4 \times 10^{-16}\)

- Solutions for further improvement
  - Coating designed for lower noise
  - Longer cavities
  - Increased spot size \(w_0\)
  - Lower temperature (note however: improvement is not necessarily monotonic with temperature)
  - Use other materials (ex: crystalline material like silicon)
  - Forget about FP cavities and use other concepts

\[H. \ J. \ Kimble \ et \ al., \ Phys. \ Rev. \ Lett. \ 101, \ 260602 \ (2008)\]
Temperature sensitivity: orders of magnitude

Many materials (BK7 glass, aluminum alloys,...) have a coefficient of thermal expansion near $\sim 10^{-5}$ K$^{-1}$

- Keeping the fractional instability $< 10^{-15}$ would require a $T^\circ$ instability $< 0.1$ nK!
- ULE glass is commonly used for both the spacer and mirror substrates
  - $T^\circ$ can be adjusted
  - $\Rightarrow$ CTE $\sim 10^{-9}$ K$^{-1}$
  - $T^\circ$ instability must be $< 1$ µK

- With Fused Silica substrates, the effective CTE is increased
  - CTE of FS: $5.5 \times 10^{-7}$ K$^{-1}$
  - 1D model is wrong, mirror is bending
  - $\Rightarrow$ Effective CTE can be $\sim 10^{-7}$ K$^{-1}$
- Can be avoided using the smart scheme of Legero et al.

T. Legero et al., arxiv 1002.2070v1
Dealing with the thermal sensitivity

Orders of magnitude: effectiveness of passive shielding

- In vacuum and with suitable design, heat transfer is dominated by radiation heat transfer. Emitted power per unit surface (Stefan-Boltzmann law): $P_{out} = \varepsilon \sigma T_e^4$

\[ W = 4\pi R^2 \varepsilon \sigma (T_e^4 - T^4) \approx 4\pi R^2 \varepsilon \sigma T_e^3 \times dT \]

Absorbed power per unit surface: $P_{in} = \varepsilon \sigma T_e^4$ ~0.19 W.K⁻¹, for $\varepsilon = 1$

Heat capacity (ULE): $H = \frac{4}{3} \pi R^3 \rho c_p$ ~843 J.K⁻¹, for $\varepsilon = 1$

- Defines a time constant of $\tau \approx 4400$ s, for $\varepsilon = 1$
- For polished, un-oxidized gold plating: $\varepsilon = 0.02$ $\Rightarrow \tau \approx 220000$ s ~2.5 days

Example of design with active stabilization and passive shielding

Cavity impulse response:
- ~2nd order LPF with $\tau > 4$ days
- $T^\circ$ coeff. ~10⁻⁷ K⁻¹

Temperature fluctuations are reduced by ~10⁵ @1000 s
Short introduction to the Pound-Drever-Hall detection

- Phase modulated laser field:

\[ E(t) = E_0 e^{-i\omega t} \]

\[ V(t) = V_0 \sin[\Omega t] \]

\[ E_m(t) = E_0 e^{-i(\omega t + \beta \sin[\Omega t])} = E_0 \sum_{n=-\infty}^{n=+\infty} J_n(\beta) e^{-i(\omega t + n\Omega t)} \]

\[ P \propto |E_m(t)|^2 = E_0^2 \sum_{n=-\infty}^{n=+\infty} \sum_{n'=-\infty}^{n'=+\infty} J_n(\beta) J^*_n(\beta) e^{-(n-n')\Omega t} = E_0^2 \]

- No intensity modulation due to cancellation of all beat notes between sidebands at \( \Omega, 2\Omega, 3\Omega, \ldots \)

- Pound-Drever-Hall detection

- In the reflected signal, near resonance, the balance between the sidebands is destroyed and an intensity modulation at \( \Omega \) is induced.

- In this method, the speed of the servo loop is NOT limited by the line width of the cavity \( \Rightarrow \) Allows tight (fast) locking to high finesse cavity.
Other things to care about

- Fluctuations of pressure
- Fluctuations of radiation pressure
- Electronic noises and environmental sensitivity of electronics
- Electronic offset + fluctuations of laser power
- Photon shot noise
- Stray etalon and fluctuations of optical path length
- Residual Amplitude Modulation in the phase modulator and its fluctuations
- Fluctuations of input beam pointing
- Interaction with other cavity modes
- Higher order modulation sidebands interacting with other cavity modes
- ...

Most of these effects are reduced in the first place by using cavities with very finesse (close to $10^6$)
  - Still they need to be considered carefully
Schematic view of laser comparison system

- Fiber Laser
- AOM
- VCO
- P.D.H. Detection
- EOM
- P.D.H. Detection
- FFT Analyzer
- Frequency counter
- FFT Analyzer

- Hg Lab
- OPUS Lab
- Finesse 800000
  \[ \lambda = 1062.5 \text{nm} \]
Comparison between two ultra stable lasers: Stability

Hg system: shields + T° stabilization. OPUS system: shields only

Proper thermal design can accommodate the higher sensitivity of fused silica mirrors

Assuming equal contributions of both laser to the flicker floor, we find the flicker floor of 1 laser at $4 \times 10^{-16}$, only possible with FS mirrors

 linewidth of $\sim 170$ mHz @1062.5 nm

High reliability: continuous operation for month

Red curve: Optical to optical comparison Hg against OPUS

Blue curve: Hg against ultra stable microwave reference through femtocomb

Drift rate $\sim -50$ mHz/s at 1062.5 nm. Fractional drift rate $\sim -1.8 \times 10^{-16}$ s$^{-1}$.


Comparison between two ultra stable lasers: Frequency noise and Phase noise power spectral density

- Frequency noise PSD

- Phase noise PSD

Blue line: calculated thermal noise limit

-19 dB[rad²/Hz] @1 Hz for 2 lasers
Conclusions and future directions

- Importance: laser noise limits the stability of most optical clocks
  - due to the Dick effect

- Already mentioned modifications of ultra stable Fabry-Perot cavity
- Lasers operating on narrow atomic lines
  
  *Active Optical Clock*

  *Prospects for a Millihertz-Linewidth Laser*
  D. Meiser et al., PRL 102, 163601 (2009)

- Stabilization to fiber delay lines
  
  *An agile laser with ultra-low frequency noise and high sweep linearity*

  *Ultralow-frequency-noise stabilization of a laser by locking to an optical fiber-delay line*
Measurement of Optical Frequencies with Optical Frequency Combs
Beat note between two lasers

Laser 2

Laser 1

Beam splitter

Photodiode

Load

Electric field: \( E(t) = E_1 \cos(\omega_1 t - \phi_1) + E_2 \cos(\omega_2 t - \phi_2) \)

Poynting vector:
\[
\Pi(t) \propto |E(t)|^2 = E_1^2 \cos^2(\omega_1 t - \phi_1) + E_2^2 \cos^2(\omega_2 t - \phi_2) + 2E_1E_2 \cos(\omega_1 t - \phi_1) \cos(\omega_2 t - \phi_2)
\]
\[
= \frac{E_1^2}{2} \left[ 1 + \cos(2\omega_1 t - 2\phi_1) \right] + \frac{E_2^2}{2} \left[ 1 + \cos(2\omega_2 t - 2\phi_2) \right] + E_1E_2 \left[ \cos((\omega_1 - \omega_2)t - (\phi_1 - \phi_2)) + \cos((\omega_1 + \omega_2)t - (\phi_1 + \phi_2)) \right]
\]

Average optical power over detector time constant:
\[
P_{opt}(t) \propto E_1^2 + E_2^2 + 2E_1E_2 \cos((\omega_1 - \omega_2)t - (\phi_1 - \phi_2)) \]

Beat note between the 2 lasers at frequency:
\[
|\omega_1 - \omega_2| \ll \omega_1, \omega_2
\]

Current through the load:
\[
I(t) = \eta P_{opt}(t)
\]

Beat note RF power:
\[
P_{RF} = \langle R_{load} \times I^2(t) \rangle \propto R_{load} \times \eta^2 |E_1|^2 |E_2|^2 \propto R_{load} \times \eta^2 P_{1opt} P_{2opt}
\]
Beat note between two lasers

- Fast photodiodes and electronics are typically limited to <100 GHz.
- 100 GHz = 0.1 THz is a tiny fraction UV/VIS/IR range (~1000 THz).
- In practice beat note measurements are **narrow band** at the scale of the UV/VIS/IR spectrum.

- Detecting a beat note is similar to using a mixer in the RF or microwave domain. However, a mixer usually gives access to both \( \omega_1 - \omega_2 \) and \( \omega_1 + \omega_2 \). Mixers are broadband devices at the scale of the RF or microwave domain.

- Other methods to “move” in the optical frequency domain:
  - Second Harmonic Generation, Sum and Difference Frequency Generation.
  - Generally **narrow band** (crystals, coatings, geometry,... specific to a narrow wavelength range).
  - Metal Insulator Metal Diode used as mixer, harmonic mixer,...
  - Broadband but practically **very tedious**.

- No straightforward way to link optical frequency to microwave frequency.
Optical Frequency Measurements prior to Optical Frequency Combs

- Harmonic frequency chain: Ensemble of subsystems each allowing a frequency multiplication by a factor 2 or 3.
  - Complexity.
  - Inconvenient frequencies (THz, mid-IR).
  - 3 or 4 chains in the world at PTB, SYRTE, NRC, JILA.
  - Continuous operation <3h.
- The setup has to be redesigned if one changes the frequency to be measured.

H. Schnatz et al. PRL 76, 18 (1996)
Optical Frequency Measurements with Optical Frequency Combs

- Harmonic chain replaced with: 1 Laser

- Extremely simple and cost effective in comparison.
- No inconvenient frequency.
- Measurement of several frequencies at the same time.
- Continuous operation for weeks.
- Covers the visible-near IR range.
- Commercially available systems.

Nobel Lecture: Defining and measuring optical frequencies
J. L. Hall, Rev. Mod. Phys. 78, 1279 (2006)

Nobel Lecture: Passion For Precision
Output of a mode-locked Laser

Ideal train of light pulses in time domain

\[ E(t) \]

\[ T_R = 1/f_R \]

- \( f_R \): repetition rate of the light pulses
- Pulse train inside the laser cavity with: \( v_g \neq v_\phi \)

Carrier envelop phase slip

Frequency domain

- Components are phase coherent
- Fourier limit: \( \Delta T \Delta v \geq 1/4\pi \)
- Comb shifted by \( f_0 \)

\[ f_0 = f_r \times \frac{\Delta \phi}{2\pi} \]
Requirements for mode-locked operation

- Gain medium with broad gain curve to support a large number of modes

- Controlled dispersion to allow short pulses to remain short through round trip propagation in the cavity
  - Chirped mirrors or prism pairs to compensate normal dispersion

- Mechanism for (passive) mode-locking: Coupling between longitudinal modes so that they oscillate with a well defined phase with respect to each others (Intra cavity non linear effect: Saturable absorber, Kerr lens)
Example of a femtosecond Ti:Sa laser

- Gain medium is a Titanium doped sapphire crystal.
- Pumped with ~6 W of laser light at 532 nm.
- Cavity made of 6 chirped mirrors.
- Intra cavity wedge for coarse tuning $f_0$.
- Fast and slow piezo actuators.
- Fast pump power control through an AOM.
- 30 fs pulses at 850 nm. Repetition rate: ~760 MHz.
- ~500 mW output power.
Femtosecond lasers used as comb generators

- **Ti:Sa**

- **Fiber lasers**
  - Erbium doped fiber
  - Ytterbium doped fiber

- **Chromium Forsterite**

- **Diode pumped Yb:KYW**

- **Microresonators**
  - Note: here the comb generation mechanism is different
    - (4-wave mixing mediated by Kerr non-linearity)
  - Repetition rate is very high

- Note: Ti:Sa and fiber femtosecond lasers well suited for optical frequency metrology are commercially available, as well as full optical frequency measurement systems
Measuring the comb parameters $f_R$ and $f_0$

Measurement of $f_R$: detect the pulse train with a fast photodiode
- Note: gives directly access to harmonics of $f_R$

Measurement of $f_0$ with an octave spanning comb: The self-referencing method
- Note: many modes contribute to the signal

Generation of an octave spanning optical frequency comb

- Short input pulses with high peak power
- Small core: high peak intensity
- Pulses remain short through propagation
- → Large non-linear effects

- Time domain: Self phase modulation, cross-phase modulation,...
- Frequency domain: Four wave mixing

- Coherent spectral broadening
  - Note: pulses are broadened → Spectral coherence but no Fourier limited pulses
Octave spanning optical frequency combs

Ti:Sa, Er-doped fiber broadened with photonic crystal fiber


- **Ultra broadband Ti:Sa**

- **Toroidal micro resonators**
  P. Del’Haye et al., arxiv 0912.4890v1 (2009)
The Optical Frequency Comb can be used in many different ways with pros and cons depending on the situation. For instance:

- $f_R$ can be locked to the RF reference
- $f_0$ can be stabilized to a reference, $f_b$ and $f_0$ can be counted and $f_0$ subtracted from $f_b$ digitally...
- In optical frequency ratio measurements, $f_R$ can be eliminated from the measurement electronically in a smart combination of $f_{b1}$ and $f_{b2}$...

Absolute frequency measurement of a Sr lattice clock

Sr lattice clock vs atomic fountain

- >15 days of ~continuous operation of the full metrological chain from optical clock to PFS
- Overall stability <6x10^{-14} @1s. Best stability for comparing a microwave clock to an optical clock.
- Overall fractional uncertainty of the measurement 2.6x10^{-15}.

Overview of all Sr measurements
- validation of the “non-perturbing” lattice scheme

\[ \nu_{87\text{Sr}} = 429\,228\,004\,229\,873.6(1.1) \text{ Hz} \]


Some other recent absolute frequency measurements


\(^{171}\text{Yb}^+\) quadrupole  Tamm et al., PRA 80, 043403 (2009)


\(^{40}\text{Ca}^+\)  M. Chwalla et al., Phys. Rev. Lett. 102, 023002 (2009)


\(^{171}\text{Yb}\)  N. D. Lemke et al., PRL 103, 063001 (2009)

\(^{40}\text{Ca}\)  G. Wilpers et al., Metrologia 44, 146 (2007)


\(^{199}\text{Hg}, ^{201}\text{Hg}\)  M. Petersen et al., Phys. Rev. Lett. 101, 183004 (2008)

Note: \(^{27}\text{Al}^+\)
Frequency ratio of Al$^+$ and Hg$^+$ single ion clocks

T. Rosenband et al., Science 319, 1808 (2008)

$\nu_{Al^+}/\nu_{Hg^+}$ is 1.052871833148990438(55)

Fractional uncertainty: $5.2 \times 10^{-17}$

<table>
<thead>
<tr>
<th>Shift</th>
<th>$\Delta \nu_{Al}$</th>
<th>$\sigma_{Al}$</th>
<th>$\Delta \nu_{Hg}$</th>
<th>$\sigma_{Hg}$</th>
<th>Limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micromotion</td>
<td>$-20$</td>
<td>20</td>
<td>$-4$</td>
<td>4</td>
<td>Static electric fields</td>
</tr>
<tr>
<td>Secular motion</td>
<td>$-16$</td>
<td>8</td>
<td>$-3$</td>
<td>3</td>
<td>Doppler cooling</td>
</tr>
<tr>
<td>Blackbody radiation</td>
<td>$-12$</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>DC polarizability</td>
</tr>
<tr>
<td>313-nm Stark</td>
<td>$-7$</td>
<td>2</td>
<td>$-7$</td>
<td>--</td>
<td>Polarizability, intensity</td>
</tr>
<tr>
<td>DC quadratic Zeeman</td>
<td>$-453$</td>
<td>0.5</td>
<td>$-1130$</td>
<td>5</td>
<td>B-field calibration</td>
</tr>
<tr>
<td>AC quadratic Zeeman</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>10</td>
<td>Trap RF B-fields</td>
</tr>
<tr>
<td>Electric quadrupole</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>10</td>
<td>B-field orientation</td>
</tr>
<tr>
<td>First-order Doppler</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>Statistical imbalance</td>
</tr>
<tr>
<td>Background gas collisions</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>4</td>
<td>Collision model</td>
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<tr>
<td>AOM phase chirp</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>6</td>
<td>RF power</td>
</tr>
<tr>
<td>Gravitational red-shift</td>
<td>$-5$</td>
<td>1</td>
<td>--</td>
<td>--</td>
<td>Clock height</td>
</tr>
<tr>
<td>Total</td>
<td>$-513$</td>
<td>23</td>
<td>$-1137$</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>
Applications: Ultra low noise microwave generation
Other Applications of Optical Frequency Combs

- **Test of comb accuracy**
  
  *Ultraprecise Measurement of Optical Frequency Ratios*
  

  *Optical clockwork with an offset-free difference-frequency comb: accuracy of sum- and difference-frequency generation*
  

- **Ultra low noise microwave generation**
  
  *The Stability of an Optical Clock Laser Transferred to the Interrogation Oscillator for a Cs Fountain*
  

  *Ultralow noise microwave generation with fiber-based optical frequency comb and application to atomic fountain clock*
  

- **Astrocombs: Use of OFC as reference for high resolution spectrometers used in astronomical telescopes**
  

- **Direct Frequency Comb Spectroscopy**
  

- **Molecular fingerprinting**
  
  *Molecular fingerprinting with the resolved modes of a femtosecond laser frequency comb, S.A. Diddams et al., Nature 445, 627 (2007)*

- **Extension to the DUV/XUV range**
  
  *A deep-UV optical frequency comb at 205 nm, Peters, E. et al., Optics Express 17, 9183 (2009)*
Optical Links
Toward an optical link across Europe (1)

- Principle: heterodyne Michelson interferometer to detect optical path length fluctuation in the fiber
- Demonstration of ultra stable optical carrier transfer in telecom fiber over ~100-200 km demonstrated in several groups (PTB, JILA/NIST, SYRTE/LPL, NMIJ/Tokyo Univ.,...)
- Over a single such segment stability is <10^{-19} @1d, when satellite systems are at ~10^{-15} @1d

Allan Deviation $\sigma_y(\tau)$

H. Jiang et al., JOSA B 25, 2029 (2008)
Demonstration of compatibility with internet traffic

- Develop repeater stations which meet reliability, robustness, cost requirements.
- Develop appropriate collaborations,... to access fibers, spaces for repeater stations,... over the relevant ~1000 km paths.

Application of Atomic Clocks to Testing the Stability of Fundamental Constants
Atomic transitions and fundamental constants (1)

- Atomic transitions and fundamental constants
  - Hyperfine transition
    \[ \nu_{\text{hfs}}^{(i)} \approx R_\infty c \times A_{\text{hfs}}^{(i)} \times g^{(i)} \left( \frac{m_e}{m_p} \right) \alpha^2 F_{\text{hfs}}^{(i)}(\alpha) \]
  - Electronic transition
    \[ \nu_{\text{elec}}^{(i)} \approx R_\infty c \times A_{\text{elec}}^{(i)} \times F_{\text{elec}}^{(i)}(\alpha). \]
  - See also, molecular vibrational and rotation => \((m_e/m_p)^{1/2}, m_e/m_p\)

- Actual measurements: ratio of frequencies

- Electronic transitions test \(\alpha\) alone (electroweak interaction)
- Hyperfine and molecular transitions bring sensitivity to the strong interaction
Atomic transitions and fundamental constants (2)

- $m_p$, $g^{(i)}$ are not fundamental parameters of the Standard Model

- $m_p$, $g^{(i)}$, can be related to fundamental parameters of the Standard Model ($m_q/\Lambda_{\text{QCD}}$, $m_s/\Lambda_{\text{QCD}}$, $m_q=(m_u+m_d)/2$)

  \[
  \frac{\delta(m_s/\Lambda_{\text{QCD}})}{(m_s/\Lambda_{\text{QCD}})} = \frac{\delta(m_q/\Lambda_{\text{QCD}})}{(m_q/\Lambda_{\text{QCD}})}
  \]

  It is often assumed that:

  V. V. Flambaum et al., PR D69, 115006 (2004)

- Recent accurate calculations have been done for some relevant transitions

  V. V. Flambaum and A. F. Tedesco, PR C73, 055501 (2006)

- Any atomic transition $(i)$ has a sensitivity to one particular combination of only 3 parameters ($\alpha$, $m_e/\Lambda_{\text{QCD}}$, $m_q/\Lambda_{\text{QCD}}$)

  \[
  \delta \ln \left( \frac{\nu^{(i)}}{R_\infty c} \right) \simeq K^{(i)}_\alpha \times \frac{\delta \alpha}{\alpha} + K^{(i)}_q \times \frac{\delta(m_q/\Lambda_{\text{QCD}})}{(m_q/\Lambda_{\text{QCD}})} + K^{(i)}_e \times \frac{\delta(m_e/\Lambda_{\text{QCD}})}{(m_e/\Lambda_{\text{QCD}})}
  \]
Sensitivity coefficients of some transitions

<table>
<thead>
<tr>
<th></th>
<th>$\kappa_\alpha$</th>
<th>$\kappa_q$</th>
<th>$\kappa_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rb hfs</td>
<td>2.34</td>
<td>-0.064</td>
<td>1</td>
</tr>
<tr>
<td>Cs hfs</td>
<td>2.83</td>
<td>-0.039</td>
<td>1</td>
</tr>
<tr>
<td>H opt</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Yb$^+$ opt</td>
<td>0.88</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Hg$^+$ opt</td>
<td>-3.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Dy comb.</td>
<td>1.5 x 10$^7$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$K_\alpha$, $K_e$ : accuracy at the percent level or better

$K_q$ : accuracy ?

<table>
<thead>
<tr>
<th>Atom</th>
<th>$^{87}$Rb</th>
<th>$^{133}$Cs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method A</td>
<td>-0.074</td>
<td>0.127</td>
</tr>
<tr>
<td>Method B</td>
<td>-0.056</td>
<td>0.044</td>
</tr>
<tr>
<td>Method C</td>
<td>-0.016</td>
<td>0.009</td>
</tr>
</tbody>
</table>

PR C73, 055501 (2006)

Dysprosium : RF transition between 2 accidentally degenerated electronic states

Dzuba et al., PRL 82, 888 (1999)

In some diatomic molecules: cancellation between hyperfine and rotational energies also leads to large (2-3 orders of magnitude enhancement)

Flambaum, PRA 73, 034101 (2006)
Overview of recent measurements

\[ \frac{d}{dt} \ln \left( \frac{\nu_{\text{th}}}{\nu_{\text{Cs}}} \right) = (-3.2 \pm 2.3) \times 10^{-16} \, \text{yr}^{-1} = -0.49 \frac{d}{dt} \ln (\alpha) - 0.025 \frac{d}{dt} \ln (m_q/\Lambda_{\text{QCD}}) \]

\[ \frac{d}{dt} \ln \left( \frac{\nu_{\text{H}}}{\nu_{\text{Cs}}} \right) = (-32 \pm 63) \times 10^{-16} \, \text{yr}^{-1} = -2.83 \frac{d}{dt} \ln (\alpha) + 0.039 \frac{d}{dt} \ln (m_q/\Lambda_{\text{QCD}}) - \frac{d}{dt} \ln (m_e/\Lambda_{\text{QCD}}) \]

\[ \frac{d}{dt} \ln \left( \frac{\nu_{\text{Sr}}}{\nu_{\text{Cs}}} \right) = (-7 \pm 18) \times 10^{-16} \, \text{yr}^{-1} = -2.74 \frac{d}{dt} \ln (\alpha) + 0.039 \frac{d}{dt} \ln (m_q/\Lambda_{\text{QCD}}) - \frac{d}{dt} \ln (m_e/\Lambda_{\text{QCD}}) \]

\[ \frac{d}{dt} \ln \left( \frac{\nu_{\text{Hg}^+}}{\nu_{\text{Cs}}} \right) = (3.7 \pm 3.9) \times 10^{-16} \, \text{yr}^{-1} = -6.03 \frac{d}{dt} \ln (\alpha) + 0.039 \frac{d}{dt} \ln (m_q/\Lambda_{\text{QCD}}) - \frac{d}{dt} \ln (m_e/\Lambda_{\text{QCD}}) \]

\[ \frac{d}{dt} \ln \left( \frac{\nu_{\text{Dy}}}{\nu_{\text{Cs}}} \right) = (-4 \pm 3.9) \times 10^{-8} \, \text{yr}^{-1} = 1.5 \times 10^{-7} \frac{d}{dt} \ln (\alpha) + 0.039 \frac{d}{dt} \ln (m_q/\Lambda_{\text{QCD}}) - \frac{d}{dt} \ln (m_e/\Lambda_{\text{QCD}}) \]

\[ \frac{d}{dt} \ln \left( \frac{\nu_{\text{Hg}^+}}{\nu_{\text{Al}^+}} \right) = (4.6 \pm 6.7) \times 10^{-17} \, \text{yr}^{-1} = -2.9 \frac{d}{dt} \ln (\alpha) \]

MPQ + LNE-SYRTE (PRL 2004)

Tokyo, JILA, LNE-SYRTE, (PRL 2008)

NIST, (PRL 2007)

Berkeley, (PRL 2007)


INDEPENDENT OF COSMOLOGICAL MODELS

Least squares fit

\[ \frac{d}{dt} \ln (\alpha) = (-0.18 \pm 0.23) \times 10^{-16} \, \text{yr}^{-1} \]

\[ \frac{d}{dt} \ln (m_q/\Lambda_{\text{QCD}}) = (131 \pm 92) \times 10^{-16} \, \text{yr}^{-1} \]

\[ \frac{d}{dt} \ln (m_e/\Lambda_{\text{QCD}}) = (3.8 \pm 5.3) \times 10^{-16} \, \text{yr}^{-1} \]
Several optical clock transitions are recognized by BIPM as secondary representations of the SI second.

In the future, optical clocks will offer a large number of possibilities for a redefining the SI second and testing fundamental physical laws.