From LEP to LHC

1. Physics of the Z boson (I)
2. Physics of the Z boson (II)
3. Physics of the W boson
4. Physics of the top quark
5. Tests of the Standard Model
6. Search for the Higgs boson (I)
7. Search for the Higgs boson (II)

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From LEP to LHC

1. Physics of the Z boson (I)
   - LEP (Statistics interlude)

2. Physics of the Z boson (II)
   - LEP (Statistics interlude)

3. Physics of the W boson
   - LEP, Tevatron, LHC (Statistics interlude)

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   - Tevatron, LHC

5. Tests of the Standard Model
   - LEP, Tevatron (Statistics interlude)

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   - LEP (Statistics interlude)

7. Search for the Higgs boson (II)
   - Tevatron, LHC

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Some bibliographic references

LEP:
arXiv:hep-ex/0511027

TEVATRON:
arXiv:0808.0147

LHC:
CMS NOTE 2006/061
Physics of the W boson

W bosons at LEP
  Production
  Couplings
  Mass measurement

W bosons at the TEVATRON
  Production
  Mass measurement

W bosons at the LHC
  Outlook for the mass measurement
Three good reasons to study W bosons

• Its mass is a free parameter of the Standard Model
• Its couplings give a direct insight to the non-Abelian structure of the Standard Model
• “Higgsometry”
LEPII, LEP2, LEP200

• From 1995 to 2000
• Replace copper RF cavities with superconducting ones to increase the beam energy
LEP Integrated Luminosity
W-boson production at LEP

Expect about 10000 W-pair/experiment

\[ \operatorname{Br}(W \rightarrow lv) = 32.5\% \]
\[ \operatorname{Br}(W \rightarrow qq) = 67.5\% \]

Measurement of the WW cross section

No ZWW vertex

Standard Model with ZWW vertex

Exerimental proof of non-Abelian structure of the Standard Model!
Life is not as easy as you’d like it to be (II)

Big calculation + Monte Carlo simulation effort to get the best out of the LEP data
W-boson couplings

\[ i \mathcal{L}^{WWV} = g_{WWV} \left[ g_Y \left( W^\dagger_{\mu\nu} W^{\mu\nu} - W^\dagger_{\mu} V_{\nu} W^{\mu\nu} \right) + \kappa_V W^\dagger_{\mu} W_{\nu} V^{\mu\nu} + \frac{\lambda_V}{m_W^2} W^\dagger_{\rho\nu} W^{\rho\nu} V^{\mu\nu} + \mathcal{C} + \mathcal{P} + \mathcal{C} \mathcal{P} + \text{dim} \geq 6 \right] \]

\[ V = \gamma, Z, \quad W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu, \quad V_{\mu\nu} = \partial_\mu V_\nu - \]

In the Standard Model:

\[ g_1^Z = g_1^\gamma = \kappa_Z = \kappa_\gamma = 1; \quad \lambda_\gamma = \lambda_Z = 0 \]

\[ g_{WWZ} = e \cot \theta_W; \quad g_{WW\gamma} = e \]

Multipole expansion of the W-\( \gamma \) interaction:

\[ Q_W = e g_1^\gamma \quad \text{Charge} \]

\[ \mu_W = \frac{e}{2m_W} \left( g_1^\gamma + \kappa_\gamma + \lambda_\gamma \right) \quad \text{Magnetic dipole} \]

\[ q_W = -\frac{e}{m_W^2} (\kappa_\gamma - \lambda_\gamma) \quad \text{Electric quadrupole} \]
W-boson couplings

Five parameters to study deviations:
\[ \Delta g_1^Z \equiv (g_1^Z - 1), \quad \Delta \kappa_\gamma \equiv (\kappa_\gamma - 1), \quad \Delta \kappa_Z \equiv (\kappa_Z - 1), \lambda_\gamma, \lambda_Z \]

Reduced to three by gauge invariance:
\[ \lambda_Z = \lambda_\gamma \quad \Delta \kappa_Z = \Delta g_1^Z - \Delta \kappa_\gamma \tan^2 \theta_W \]

Measure \( g_1^Z, \kappa_\gamma, \lambda_\gamma \)
Measurement of WWZ and WWγ couplings

The production angle of the W⁻ boson carries information on the production mechanism and therefore the coupling.
Measurement of WWZ and WW$\gamma$ couplings

- Standard Model
- 68% C.L., 1-par fit
- 2-par fit
- 68% C.L., 2-par fit
- 95% C.L., 2-par fit
- 3-par fit
- 68% C.L., 3-par fit proj
Threshold measurement of $m_W$

$W$-boson pair-production cross section at threshold depends on the centre-of-mass energy and on $m_W$

\[ \sigma_{WW} = \sigma(M_W, \sqrt{s}) \]

Measure the cross section to extract $m_W$

\[ \sigma_{WW} = 3.69 \pm 0.45 \text{ pb} \]
\[ M_W = 80.40 \pm 0.22 \text{ GeV} \]

Not very efficient:
- low statistics
- low sensitivity
- impatient Higgs-hunters
Direct determination of $m_W$

- A “smarter” version of measuring the invariant mass of the decay products

- Apply kinematic fits:
  - In $qqqq$ events nothing is lost!
    4C Conserve $(p, E)$ 5C Two $W$ bosons of equal mass
  - In $qqlv$ events a massless particle is lost
    1C Missing mass=0 2C Two $W$ bosons of equal mass

Need to control the centre-of-mass energy

![Graphs showing before and after fit](image)
Measured $m_W$ spectra
Fitting $m_W$

- Prepare Monte Carlo simulation of all small detector imperfections
- Generate Monte Carlo events for a given $m_W$
- “Reweight” these events to emulate any possible value of $m_W$
- Compare data and Monte Carlo to find the best-fitting value of $m_W$

$w_i(m'_w) = \frac{|M(m'_w)|^2}{|M(m_w)|^2}$
Main systematic effects from QCD

- **Measure jets and not quarks.**
- **Are the invariant mass of jets really related to $m_W$?**
  - **Hadronisation**
    The way quarks generate visible particles
  - **Bose-Einstein correlations**
    Phase-space of bosons know about other (identical) bosons
  - **Colour Reconnections**
    Hadrons from one $W$ can “talk” to hadrons from the other $W$
Invent ways to reduce systematic uncertainties.

Example: Colour Reconnections

Colour Reconnection affects low-energy particles.

Remove all particles with an energy below a given threshold.

Optimal cut at 2 GeV as tradeoff between statistical and systematic uncertainties.
**LEP Results**

$$m_W (\text{LEP}) = 80.376 \pm 0.033 \text{ GeV/c}^2$$

weights: $qqqq = 22\%$, $qqlv = 76\%$, $\sigma = 2\%$

(prob. fit.: $11.1\%$)

**Difference between channels:**

$$m_W(4q-2q) = -11.7 \pm 44.6 \text{ MeV/c}^2$$
## Uncertainties on $m_W$ at LEP

<table>
<thead>
<tr>
<th>Source</th>
<th>Systematic Error on $m_W$ (MeV)</th>
<th>(q\bar{q}\ell\nu_\ell)</th>
<th>(q\bar{q}qq)</th>
<th>Combined</th>
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</thead>
<tbody>
<tr>
<td>ISR/FSR</td>
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<td>8</td>
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<tr>
<td>Hadronisation</td>
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<td>Detector Systematics</td>
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<td>LEP Beam Energy</td>
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<tr>
<td>Colour Reconnection</td>
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<td>8</td>
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<tr>
<td>Bose-Einstein Correlations</td>
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<td>Other</td>
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<td>Total Systematic</td>
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<td>40</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
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<td>59</td>
<td>33</td>
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<tr>
<td>Statistical in absence of Systematics</td>
<td></td>
<td>30</td>
<td>27</td>
<td>20</td>
</tr>
</tbody>
</table>
Contributions to the total uncertainty

- Hadronisation
  - 17.5%
  - 7.6%
  - 18.4%
  - 56.5%

- Detector
  - 4.6%
  - 7.2%
  - 5.5%
  - 1.2%

- Statistics

- EW radiation
- Beam E
- CR & BEC
- Others
Statistical Interlude

Combining correlated measurements of several different physical quantities

BLUE technique

Valassi NIM A500 (2003) 391
Definitions

\[ X_\alpha = (X_1, \ldots, X_N) \quad \text{N true values} \]

\[ y_i = (y_1, \ldots, y_n) \quad \text{n}_\alpha \text{ measurements of each } X_\alpha \]

\[ n = \sum_\alpha n_\alpha \geq N \]

Associate measurements and sought values

\[(n \times N) \text{ matrix} \quad \mathcal{U}_{i\alpha} = \begin{cases} 
1 & \text{if } y_i \text{ is a measurement of } X_\alpha, \\
0 & \text{if } y_i \text{ is not a measurement of } X_\alpha.
\end{cases} \]

Covariance matrix

\[ M_{ij} = \text{cov}(y_i, y_j) = \text{cov}(y_j, y_i) = M_{ji} \]

Looking for the estimates \( \hat{X}_\alpha \) of the \( X_\alpha \)
BLUE

Best

the estimates are those with minimum variance (distance from true values among all possible estimates)

Linear

the estimates are built as linear combinations of the input measurements

Unbiased

the expectation value of the estimates is the true value
BLUE

Best
the estimates are those with minimum variance (distance from true values among all possible estimates)

Linear
the estimates are built as linear combinations of the input measurements

\[ \hat{x}_\alpha = \sum_{i=1}^{n} \lambda_{\alpha i} y_i = \sum_{\beta=1}^{N} \sum_{i=1}^{n} \lambda_{\alpha i} U_{i \beta} y_i. \]

Unbiased
the expectation value of the estimates is the true value

\[ \sum_{i=1}^{n} \lambda_{\alpha i} U_{i \beta} = \delta_{\alpha \beta} \quad \forall \alpha, \forall \beta. \]
The technique

Covariance matrix of the estimates:

\[
\text{cov}(\hat{x}_\alpha, \hat{x}_\beta) = \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_{\alpha i} M_{ij} \lambda_{\beta j} \quad \text{from} \quad M_{ij} = \text{cov}(y_i, y_j), \quad \hat{x}_\alpha = \sum_{i=1}^{n} \lambda_{\alpha i} y_i
\]

Diagonal elements of the covariance matrix:

variances (square of uncertainties)

\[
\text{var}(\hat{x}_\alpha) = \text{cov}(\hat{x}_\alpha, \hat{x}_\alpha) = \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_{\alpha i} M_{ij} \lambda_{\alpha j}
\]

minimised by

\[
\lambda_{\alpha i} = \sum_{\beta=1}^{N} (\tilde{U} M^{-1} U)_{\alpha \beta}^{-1} (\tilde{U} M^{-1})_{\beta i}
\]

giving the solution through

\[
\hat{x}_\alpha = \sum_{i=1}^{n} \lambda_{\alpha i} y_i
\]
Uncertainties

• The covariance matrix of the estimates is

\[ \text{cov}(\hat{x}_\alpha, \hat{x}_\beta) = (\tilde{U} M^{-1} U)_{\alpha\beta}^{-1} \]

(and the roots of the diagonal elements are the uncertainties on the estimates)

• Separate sources of uncertainties are easy to handle.

\[ M_{ij} = \text{cov}(y_i, y_j) = \sum_{u=1}^{U} \text{cov}^{[u]}(y_i, y_j) = \sum_{u=1}^{U} M_{ij}^{[u]} \]

Propagate to final result

Separate different contributions

\[ \text{cov}(\hat{x}_\alpha, \hat{x}_\beta) = \sum_{u=1}^{U} \text{cov}^{[u]}(\hat{x}_\alpha, \hat{x}_\beta) \]

\[ \text{cov}^{[u]}(\hat{x}_\alpha, \hat{x}_\beta) = \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_{\alpha i} M_{ij}^{[u]} \lambda_{\beta j} \]
Goodness of fit

There is no explicit $\chi^2$ minimization.

Goodness of fit can be assessed by constructing the distance of the measurements from the correspondent linear estimates

$$\sum_{\alpha=1}^{N} \sum_{\beta=1}^{N} \sum_{i=1}^{n} \sum_{j=1}^{n} [U_{i}\alpha(y_{i} - \hat{x}_{\alpha})] M^{-1}_{ij}[U_{j}\beta(y_{j} - \hat{x}_{\beta})]$$

Minimum value of the distance is distributed as a $\chi^2$ (if all errors are Gaussian) with $n-N$ degrees of freedom and can be used to assess the goodness of fit.
Advantages of the BLUE technique

• Simplicity: calculation of the weights once leads to both estimates and their uncertainties

• Intuitive handling of multiple sources of uncertainty. Well suited to HEP-style combinations with correlated/uncorrelated uncertainties.

• BUT large correlations make the technique unstable. Difficult matrix inversion, large weights, dangerous fluctuations. An ill-posed problem?
Uncertainties on the boson masses

\[ m_Z = 91.1875 \pm 0.0021 \text{ GeV} \quad (23 \text{ ppm}) \]
\[ m_W = 80.376 \pm 0.033 \text{ GeV} \quad (410 \text{ ppm}) \]

**LEP1:** 15 million Z bosons
**LEP2:** 40000 W bosons

Uncertainty on \( m_W \) statistically limited:

apart from the beam energy, systematic uncertainties are also studied with W bosons!!

**Produce more W bosons !(?)**
Features:

- Precision silicon vertexing
- Large radius drift chamber (r=1.4m)
- 1.4 T solenoid
- EM+HAD Calorimetry
- Muon chambers (|η| < 1.1)
- Particle Identification
Experiments: DZero (D0)

Features:

- Precision silicon vertexing
- Outer Fiber Tracker (r=0.5m)
- 2.0 T solenoid
- EM+HAD Calorimetry
- muon chambers (|η| < 2.0)
Transverse momentum & MET

- Momentum of colliding partons is mostly along the beam.
- The momentum in the plane transverse to the beam is conserved:
  \[ \sum_{\text{part}} p_x = 0 \quad \sum_{\text{part}} p_y = 0 \]
- Any missing momentum in the transverse plane is attributed to the neutrino
  (Or other to other non-interacting particles such as the limitinos)
- Transverse momentum:
  \[ p_T = \sqrt{p_x^2 + p_y^2} \]
Luminosities and cross sections

- About 4.5/fb delivered
- About 4.0/fb recorded

5 M events/fb⁻¹
W-boson production at the TEVATRON

Concentrate on semileptonic decays:
- trigger
- purity
- precise reconstruction
Cross-section measurement

$\sigma \times \text{Br} (W\rightarrow l\nu)$

NNLO theory curves:
Martin, Roberts, Stirling, Thorne

Run I

Run II

$E_{cm}$ (TeV)

D0 II (e)  CDF II (e+\mu)
D0 II (\mu)  CDF II (e,1.2<|\eta|<2.8)
D0 I (e)  CDF I (e)
Physics of the W boson

W bosons at LEP
  Production
  Couplings
  Mass measurement

W bosons at the TEVATRON
  Production
  Mass measurement

W bosons at the LHC
  Outlook for the mass measurement
The transverse mass

\[ m_T(l^\pm \nu) = \sqrt{2 \, p_T(l^\pm) \, p_T(\nu) \, (1 - \cos(\phi(l^\pm) - \phi(\nu)))} \]
The template method

Generate $O(1000)$ spectra for $80 \text{ GeV} < m_W < 90 \text{ GeV}$ and find the one which fits the data best

$$f(m_W)$$
Fitting sensitive variables

Maximum likelihood fits to determine $m_W$

- Transverse mass
- Electron transverse momentum
- Missing $P_T$
Main systematic uncertainties

- Modeling of W-boson production and decay
- Lepton momentum/energy scale and resolution
- Recoil energy scale and resolution
Modeling of $W$ production and decay

- **PDF of partons inside the proton**
  - change $W$ boost/distribution, change distribution of the leptons, skew final spectra
- **Photon final state radiation**
  - change lepton angles and final spectra
- **QCD initial state radiation**
  - change $W$ $p_T$ and final spectra
- **$\Gamma_W$**
  - need to know the width to predict the shape of the spectra
Resolution vs. energy scale

Needed to model the spectrum to be fitted

Needed to know what you are measuring

Measurement-Truth
Lepton momentum/energy scale

Calibrate tracker with decay of known resonances into muon pairs

CDF
Lepton momentum/energy scale

Fix electromagnetic calorimeter energy scale with $E/p$ of electrons and $Z \rightarrow ee$ mass peak
Fix hadron calorimeter energy scale

The hadronic recoil should balance the well-measured $p_T$ of Z bosons decaying into lepton pairs
Measurement of $m_w$ at CDF-run II

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$W$ boson mass ($\text{MeV}/c^2$)</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_T(e, \nu)$</td>
<td>$80493 \pm 48_{\text{stat}} \pm 39_{\text{syst}}$</td>
<td>86/48</td>
</tr>
<tr>
<td>$p_T^\ell(e)$</td>
<td>$80451 \pm 58_{\text{stat}} \pm 45_{\text{syst}}$</td>
<td>63/62</td>
</tr>
<tr>
<td>$p_T^\nu(e)$</td>
<td>$80473 \pm 57_{\text{stat}} \pm 54_{\text{syst}}$</td>
<td>63/62</td>
</tr>
<tr>
<td>$m_T(\mu, \nu)$</td>
<td>$80349 \pm 54_{\text{stat}} \pm 27_{\text{syst}}$</td>
<td>59/48</td>
</tr>
<tr>
<td>$p_T^\ell(\mu)$</td>
<td>$80321 \pm 66_{\text{stat}} \pm 40_{\text{syst}}$</td>
<td>72/62</td>
</tr>
<tr>
<td>$p_T^\nu(\mu)$</td>
<td>$80396 \pm 66_{\text{stat}} \pm 46_{\text{syst}}$</td>
<td>44/62</td>
</tr>
</tbody>
</table>

Combined result (200/pb) with BLUE technique

$m_w = 80.413 \pm 0.034$ (stat.) $\pm 0.034$ (syst.) GeV =

$m_w = 80.413 \pm 0.048$ GeV
Measurement of $m_W$ at CDF-run II

Combined result (200/pb)

$m_W = 80.413 \pm 0.034 \text{ (stat.)} \pm 0.034 \text{ (syst.)} \text{ GeV} = 

m_W = 80.413 \pm 0.048 \text{ GeV}$
Combination of Tevatron $m_W$ measurements

\[ m_W = 80.413 \pm 0.039 \text{ GeV} \]

- Uncorrelated: 39MeV
- PDF: 13MeV
- EW radiative corrections: 11MeV
- Tot. Correlated 17MeV
Correlations of Tevatron $m_W$ measurements

<table>
<thead>
<tr>
<th></th>
<th>Run-0</th>
<th>Run-I</th>
<th>Run-II</th>
</tr>
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<tr>
<td></td>
<td>CDF</td>
<td>CDF-Ia</td>
<td>CDF-Ib</td>
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<td>CDF Run-0</td>
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<td>CDF-Ia</td>
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<tr>
<td>CDF-Ib</td>
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<td>1.0</td>
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<td>DØ</td>
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</tr>
<tr>
<td>CDF-II</td>
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</tr>
</tbody>
</table>
Outlook for $m_W$ at the TEVATRON

Systematic effects assessed from data: scale with statistics!
Average of LEP and TEVATRON

All systematic uncertainties are uncorrelated!

W-Boson Mass [GeV]

- TEVATRON: $80.432 \pm 0.039$
- LEP2: $80.376 \pm 0.033$
- Average: $80.399 \pm 0.025$
- $\chi^2$/DoF: $1.2/1$

July 2008
Statistical Interlude

How good is LEP/TEVATRON

\[ \chi^2 / \text{dof} = 1.2 / 1 \]

How good is LEP

\[ \chi^2 / \text{dof} = 49 / 41 \]
Statistical Interlude

The diagram illustrates the relationship between the p-value for a test and the confidence intervals. The horizontal axis represents the chi-squared ($\chi^2$) value, while the vertical axis shows the p-value for the test. The confidence intervals are marked by the lines on the chart.

Specifically, the diagram shows:
- For $n = 1$, the p-value is approximately 1.2/1.
- For $n = 40$, the p-value is approximately 49/41.

These values indicate the significance level and confidence level for different sample sizes. The chart helps in understanding how the p-value changes with the sample size.
Statistical Interlude

\[ \chi^2/n = 1.2/1 \]

\[ 49/41 = 1.19 \]

Degrees of freedom \( n \)

- 1%
- 5%
- 10%
- 32%
- 68%
- 95%
- 99%

Salvatore Mele | From LEP to LHC | Troisième cycle
Physics of the W boson

W bosons at LEP
  Production
  Couplings
  Mass measurement

W bosons at the TEVATRON
  Production
  Mass measurement

W bosons at the LHC
  Outlook for the mass measurement
LHC as a W (and Z) factory!

\[ \sigma(pp\rightarrow W\rightarrow\nu) \sim 30 \text{nb} \]

3x10^8 W events in 10 fb^{-1}

No problems of statistics

Control systematic uncertainties in a difficult experimental and theoretical environment

\[ \sigma(pp\rightarrow Z\rightarrow ll) \sim 3 \text{nb} \]

3x10^7 Z events in 10 fb^{-1}

Take advantage of Z bosons to reduce systematic uncertainties
The challenge of systematics

- Aim to measure $m_W$ with $15\text{MeV}$ uncertainty
- Keep all source of systematics below $10\text{MeV}$
- For energy and momentum scales this means $\sim 10\text{MeV}/\sim 100\text{GeV}$ which is $10^{-4}$
- Use the Z boson sample to control systematics:
  - by fixing the energy scale (ATLAS)
  - exploring alternative methods (CMS)
“Traditional” method à la TEVATRON

Fit the transverse mass from data using a model which takes into account detector smearing

Improved procedures for control of lepton and calorimetric energy scale with Z bosons

ATLAS 10 fb⁻¹

<table>
<thead>
<tr>
<th>Source</th>
<th>Δm_W (ATLAS)</th>
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<tbody>
<tr>
<td>Statistics</td>
<td>&lt; 2 MeV</td>
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<td>E-p scale</td>
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<tr>
<td>Energy resolution</td>
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</tr>
<tr>
<td>Lepton identification</td>
<td>5 MeV</td>
</tr>
<tr>
<td>Recoil model</td>
<td>5 MeV</td>
</tr>
<tr>
<td>W width</td>
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</tr>
<tr>
<td>Parton distribution functions</td>
<td>10 MeV</td>
</tr>
<tr>
<td>Radiative decays</td>
<td>&lt; 10 MeV</td>
</tr>
<tr>
<td>p_T, W</td>
<td>5 MeV</td>
</tr>
<tr>
<td>Background</td>
<td>5 MeV</td>
</tr>
<tr>
<td>TOTAL</td>
<td>25 MeV</td>
</tr>
</tbody>
</table>
The “scaled observable” method

• Select Z-boson events, consider just one lepton and construct some observables $O^Z$ similar to those $O^W$ of W-boson events: $m_T$, scaled electron energy $E_e/m_Z$ or $p_e/m_W$

• Use the distribution measured for the Z to predict those of the W as a function of $m_W$

\[
\left. \frac{d\sigma^W}{dO^W} \right|_{\text{pred}} = \left. \frac{M_Z}{M_W} R(X) \frac{d\sigma^Z}{dO^Z} \right|_{\text{measured}} \Rightarrow O^Z = \frac{M_Z}{M_W} O^W
\]

\[
R(X) = \frac{d\sigma^W}{dX^W} / \frac{d\sigma^Z}{dX^Z} \quad \text{Extracted from theory}
\]

\[
X^V = \frac{O^V}{M^V} \quad \text{Scaled observable}
\]

• Compare $\left. \frac{d\sigma^W}{dO^W} \right|_{\text{pred}}$ with data to extract $m_W$
Move the Z prediction till it matches the W observation
Systematic uncertainties at LHC

Many systematic uncertainties are reduced by using these ratios

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>$\Delta M_W$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
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<td>Energy scale</td>
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<tr>
<td>Recoil model</td>
<td>&lt;10</td>
</tr>
<tr>
<td>Background</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total instrumental</strong></td>
<td><strong>&lt;20</strong></td>
</tr>
<tr>
<td>PDF</td>
<td>&lt;10</td>
</tr>
<tr>
<td>W width</td>
<td>&lt;15</td>
</tr>
<tr>
<td>$p_T^W$</td>
<td>&lt;30</td>
</tr>
</tbody>
</table>
Conclusions
Remarks on the measurement of $m_W$

• “Brute force” (a.k.a. statistics and centre-of-mass energy) are just one of the tools

• Limiting factor: systematic uncertainties
  - LEP: QCD
    Total 33MeV
  - TEVATRON: W production & Energy scale
    Total 39MeV, uncorrelated from LEP, combined 25MeV
  - LHC: W production
    Aim for 15-20MeV